

Statistical Analysis of Case Recovery Rate

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Abstract

This paper propose to develop statistical measures of case recovery rate assuming that patient recovery is influenced by two factors acting simultaneously, that is, to develop a two way analysis of case recovery rate for a disease. The specific rates which are useful in identifying segments of the population that are at elevated risk of a given disease and would need a more aggressive medical and health interventionist measures to alleviate the disease compared with the average number of the general population at risk were estimated. Case recovery rate has shown to possibly serve as a measure of morbidity of a disease in a population especially if the disease is not virulent. It is shown that the smaller this rate is, the more likely is the disease morbid in the population and vice versa. It is also shown under certain condition that case recovery rate when used as a measure of survivorship or morbidity from a disease must be used and interpreted with great caution because of definitional problems. The propose method was illustrated with some data.

Keywords: *Case fatality, Recovery rate, Crude rate, Adjusted and unadjusted rates, Morbidity*

Background to the Study

The concept of case fatality rate, the number of deaths due to a certain disease per infected population at risk during a specific time period is often used as a gauge of the virulence of a disease and of the success of its treatment and management strategies. However, the concept of case recovery rate have been defined as the number of persons who recovered from a certain disease per population at risk marked by the cessation of the relevant symptoms during a specified period of time. (Zar 1984, 1996; Fleiss, 1981, Cohen 1988). It may also be used as a measure of the success of the disease treatment and management program. (Crawford et al 1992, Yeates and Taylor 1997). This may be a more relevant measure especially for life threatening highly virulent severe and acute disease outbreak including AIDS/HIV and Ebola.

(Daniel 1983; Cole and MacMahon 1971; Cook, Doll. and Fellingham 1969; Hennekens and Buring 1987, Cicchetti et al 1991, 1993, Rourke, et al 1992). The prognosis of a disease is usually dependent on several factors including the demographic and socio-economic characteristics of the patient as well as the length of time that elapsed between disease onset and hospitalization and available medical and health management technologies. These variables are usually associated with the occurrence of the event of interest as well as with one another. (Kelly and Cowling 2013; Taubenberger Jeffrey and David 2006; Li et al 2008; King 2008, Cornfield 1951, Crawford, and Allan 1997).

Objective of the Study

This paper hereby examine to develop statistical measures of case recovery rate assuming that patient recovery is influenced by two factors acting simultaneously, that is, we propose to develop a two way analysis of case recovery rate for a disease.

Methodology

Let A with 'a' levels or strata and B with 'b' levels or strata be any two factors influencing the outcome of a certain disease during a specified time period. Let n_{ij} be the number of persons at the i th level of factor A and j th level of factor B in the population at risk who contacted the disease within a specified time period, for $i = 1, 2, \dots, a$ and $j = 1, 2, \dots, b$. Let f_{ij} be the corresponding number of persons who recovered from the disease within the specified period of time.. These two numbers may be written as the pair (f_{ij}, n_{ij}) . The factor A by factor B case specific recovery rate r_{ij} , at the i th level of factor A and j th level of factor B is then given

$$r_{ij} = \frac{f_{ij}}{n_{ij}}, \quad i = 1, 2, \dots, a; j = 1, 2, \dots, b \quad \dots\dots\dots 1$$

Let $n_i = \sum_{j=1}^b n_{ij}$ be the number of persons in the population at risk infected by the disease at the i th level of factor A, $n_j = \sum_{i=1}^a n_{ij}$, the number of persons in the population at risk infected at the j th level of factor B for all levels of factor A. The total number of persons in the infected population at risk of recovery or the infected population is $n = \sum_{i=1}^a n_i = \sum_{j=1}^b n_j = \sum_{i=1}^a \sum_{j=1}^b n_{ij}$

Let $p_i = \frac{n_i}{n}$ be the proportion of the population at risk infected at the i th level of factor A and $p_j = \frac{n_j}{n}$ be the proportion of the infected population at risk infected in the j th level of factor B. Also let $p_{ij} = \frac{n_{ij}}{n}$ be the proportion of the population at risk infected in the i th level of factor A and the j th level of factor B where $\sum_{i=1}^a p_i = \sum_{j=1}^b p_j = \sum_{i=1}^a \sum_{j=1}^b p_{ij} = 1$

$$\text{Let } f_i = \sum_{j=1}^b f_{ij}; \quad f_j = \sum_{i=1}^a f_{ij} \quad \dots\dots\dots 2$$

be respectively the number of persons who recovered from the disease at the i th level of factor A during the specified time period and the j th level of factor B who recovered from disease during the specified time period.

$$\text{Let } \bar{f} = \sum_{i=1}^a f_i = \sum_{j=1}^b f_j = \sum_{i=1}^a \sum_{j=1}^b f_{ij} \quad \dots\dots\dots 3$$

be the total number of persons in the population at risk who recovered from the disease during the specified time period.

Then the unadjusted factor A specific recovery rate at the *ith* level of factor A, namely

$$r_i = \frac{f_i}{n_i} = \frac{\sum_{j=1}^b f_{ij}}{n_i} = \sum_{j=1}^b \frac{p_j r_{ij}}{p_i} \quad \dots\dots\dots 4$$

Similarly, the unadjusted specific recovery rate at the

$$r_j = \frac{f_j}{n_j} = \frac{\sum_{i=1}^a f_{ij}}{n_j} = \sum_{i=1}^a \frac{p_i r_{ij}}{p_j} \quad \dots\dots\dots 5$$

Also, the overall total or crude unadjusted recovery rate from the disease during the specified time period is

$$r = \frac{f}{n} = \frac{\sum_{j=1}^b n_j r_j}{n} = \frac{\sum_{j=1}^b p_j r_j}{n} \quad \dots\dots\dots 6$$

The specific rates in equations 1-3 are useful in identifying segments of the population that are at elevated risk of a given disease and would therefore be in need of more aggressive medical and health interventionist measures to alleviate the disease compared with the average number of the general population at risk.

Further interest here is also often to obtain rates adjusted for the two factors of classification. As noted above, factors A and B are usually closely associated with the outcome of the disease as well as with each other. There are therefore likely to have confounding effects on the estimated rates. The rates standardization or adjustment is aimed at removing some of these effects. If the distribution of the population by the levels of the factors of classification is available as in the present case, then the method of standardization that readily suggests itself is the direct method.

To apply the direct method of standardization, we use the overall distribution of the population at risk over the categories of factor B namely $p_j ; j = 1, 2, \dots, b$ as the standard population for adjusting the specific rates for the levels of factor A. Similarly, we use the overall distribution of the population at risk over the levels of factor A namely $p_i ; i = 1, 2, \dots, a$, as the standard population to apply in adjusting its specific rates for the levels of factor B.

Thus, the direct standardized or adjusted recovery rate for the *ith* level of factor A namely r_i direct is calculated as

$$r_i \text{ direct} = \sum_{j=1}^b p_j r_{ij} ; j = 1, 2, \dots, b \quad \dots\dots\dots 7$$

Similarly, the direct adjusted rate for the *jth* of factor B namely r_j direct is given as

$$r_j \text{ direct} = \sum_{i=1}^a p_i r_{ij} ; i = 1, 2, \dots, a \quad \dots\dots\dots 8$$

The direct adjusted crude rate, $r_{adj. direct}$ resulting from equations 7 and 8 may be equal but are not necessarily equal to the unadjusted crude rate r obtained from equation 6 ..

The format of the calculation is presented in the following table.

Table1. Schematic Table Illustrating the Calculation of Specific and Direct Adjusted Rates in a Two - Way Classification

Factor A	1	2	3	...	b	Proportion of Popn $p_{i.}$	Unadjusted rate ($r_{i.}$)	Direct adjusted rate $r_{i.}$ Direct
1	r_{11}	r_{12}	r_{13}	...	r_{1b}	$p_{1.}$	$r_{1.}$	$r_{1.}$ direct
2	r_{21}	r_{22}	r_{23}	...	r_{2b}	$p_{2.}$	$r_{2.}$	$r_{2.}$ direct
3	r_{31}	r_{32}	r_{33}	...	r_{3b}	$p_{3.}$	$r_{3.}$	$r_{3.}$ direct
.
a	r_{a1}	r_{a2}	r_{a3}	...	r_{ab}	$p_{a.}$	$r_{a.}$	$r_{a.}$ direct
Proportion of Popn $p_{.j}$	$p_{.1}$	$p_{.2}$	$p_{.3}$...	$p_{.b}$		$r_{.j}$	
Unadjusted rate $r_{.j}$	r_{a1}	r_{a2}	r_{a3}	...	r_{ab}	$p_{a.}$	$r_{a.}$	$r_{a.}$ direct
Direct adjusted rate $r_{.j}$ Direct	$r_{.1}$ direct	$r_{.2}$ direct	$r_{.3}$ direct	...		$p_{.b}$ direct		

Table 2 : Distribution of Meningitis Patient and Number who recovered within two weeks of Treatment by Age of patients and type of Medication. The 1st entry is the number of people who recovered and the 2nd entry is the number of meningitis patients treated for 2 weeks

Age	Natural herbs	Standard Drug	New Drug	Total
< 10	5; 72	10; 130	13; 150	28; 352
10-19	5; 76	4; 59	7; 85	16; 220
20-29	6; 114	4; 70	9; 108	19; 292
30-39	3; 53	4; 49	5; 56	12; 158
40-49	4; 67	8; 105	4; 52	16; 224
50+	4; 60	5; 63	7; 78	16; 201
Total	29; 442	35; 476	45; 529	107; 1447

The age by type of treatment specific rate are obtained by applying equation 1 to table 2. The results are presented in Table 3.

Table 3: Age by Type of Treatment Meningitis Recovery Rate per 1000 Patients

Age	Natural herbs	Standard Drug	New Drug	Total
< 10	69.4	76.9	86/7	79.5
10-19	65.8	67.8	82.4	72.7
20-29	52.6	57.1	83.3	65.1
30-39	56.6	81.6	89.2	76.9
40-49	59.7	76.2	76.9	71.4
50+	66.7	5;63	89/7	79.6
Unadj type of treatment specific (r.j)	65.6	73.5	85/1	73.9 (r)

As expected, recovery is dependent on type of medication, increasing from 65. 6 for natural herbs to a high level of 35.1 per thousand patients treated with new drug. However, age specific recovery rates do not seem to have a consistent pattern with age. This finding will seem to indicate the need to develop appropriate strategies to re-orientate patients who tend to prefer use of natural herbs.

Using equation 5, we calculate the direct adjusted specific recovery rates by age groups using the distribution of patients across types of treatment as the standard population. The calculation is shown in Table 4

Table 4: Calculation of Direct Adjusted Specific Recovery Rates by Age using Distribution of Patients by Types of Treatment as Standard Population (in Thousands)

Age	Natural Herbs	Standard Drug	New Drug	Unadjusted Rate (r_i)	Direct Adjusted (r_i direct)
<< 10 r_{1j}	69.4	76.9	86.7	79.5	
$p_j r_{1j}$	21.2	25.3	31.7		78.2
10 – 19 r_{2j}	65.8	67.8	82.4	72.7	
$p_j r_{2j}$	20.1	22.3	30.2		72.6
20 – 29 r_{3j}	52.6	57.1	83.3	65.1	
$p_j r_{3j}$	16.0	18.8	30.5		65.3
30 – 39 r_{4j}	56.6	81.6	89.2	76.9	
$p_j r_{4j}$	17.3	26.8	32.6		76.8
40 – 49 r_{5j}	59.7	76.2	76.9	71.4	
$p_j r_{5j}$	18.2	25.1	28.2		71.5
50 + r_{6j}	66.7	79.4	89.7	79.6	
$p_j r_{6j}$	20.3	26.1	32.8		79.2
Total r_{tj}	65.6	73.5	85.1	73.9	
p_{6j}	20.0	24.2	31.2		75.3
Proportion of Patients in Treatment Type	0.305	0,329	0.366		

Note that adjusted of age specific recovery rate for type of treatment slightly reduces these rates but not significantly, indicating that the type of treatment the patient receives may not be importantly associated with age of patient.

Finally, the direct adjusted specific recovery rate by type of treatment using the distribution of affected patients by age as standard population is calculated from equation 6 as the results are presented in Table 5.

Table 5: Calculation of Direct Adjusted Specific Recovery Rates by Treatment using the Distribution of Patients by Age as Standard Population (in thousands)

Age	Natural Drug		Standard Drug		New Drug		Total		Proportion of Patients in Age Group (Standard Pop)
	r_{i1}	$p_i r_{i1}$	r_{i2}	$p_i r_{i2}$	r_{i3}	$p_i r_{i3}$	r_{it}	$p_i r_{it}$	
<< 10	69.4	16.9	76.9	18.7	86.7		79.5	19.3	0.243
10 – 19	65.8	10.0	67.9	10.3	82.4		72.7	11.1	0.152
20 – 29	52.6	10.6	57.1	11.5	83.3		65.1	13.2	0.202
30 – 39	56.6	6.2	81.6	8.9	89.2		76.9	8.4	0.109
40 – 49	59.7	9.3	76.2	11.8	76.9		71.4	11.1	0.155
50 +	66.7	9.3	79.4	11.0	89.7		79.6	11.1	0.139
Unadjusted Specific r_j	65.6		73.5		85.1				
Direct Adj Specific rate r_{i1}^{direct}		62.3		72.2				74.2	

It is observed that when type of treatment specific recovery rates is adjusted for age of patients, their values are slightly reduced for each treatment type, although the observed patient still remains.

Conclusion

We have in this paper proposed the concept of case recovery rate defined as the number of infected persons who recovered from a specific disease per population of persons infected with the disease within a specified time period as a public health indicator. Both the unadjusted and adjusted factor specific rates help the public health worker to determine those factors and combination of factors that would need special public health and medical attention.

As noted earlier, case fatality rate and case recovery rate may be used to measure the success of a disease management program in controlling disease. However, unlike case fatality rate, case recovery rate can also serve as a measure of morbidity of a disease in a population especially if the disease is not virulent. The smaller this rate is, the more likely is the disease morbid in the population but if the recovery rate is high, the chances that the disease is morbid may be small.

Never the less, unlike case fatality rate which has an element of finality, case recovery rate when used as a measure of survivorship or morbidity from a disease must be used and interpreted with great caution because of definitional problems. Recovery is often a continuous process that may be prolonged especially with increasing age and compounded by other associated conditions that may affect recovery. However, if recovery is identified with certain physical factors such as resumption of usual economic and social activities with the cessation of some well defined primary symptoms, then the resumption of these activities and the cessation of these symptoms may be regarded as necessary and fairly sufficient indication of recovery. Under these conditions, case recovery rate may serve as a good public health indicator.

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