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MATHEMATICAL MODEL FOR STRENGTH OF CONCRETE PRODUCED FROM 8-16MM SIZE AGGREGATES FROM UMUAGA GRAVEL PITHS

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Abstract
In this report the quality of (8-16mm size) aggregate from Umuaga which has a doubtful quality was studied through a mathematical model built from Sheffe's Simplex method of mixture design to cover a range of water/cement ratio , fine and course aggregate ratios of 0.5 to 0.6 ,1.0 to 1.5 and 1.5 to 4, respectively. The optimum strength, water/cement ratio and associated mix proportions of 12.231N/mm2 ,0.532 and 1:1.6:2.7, respectively, were obtained. When compared with the strength of a similar mix proportion with granite aggregate ,estimated at about 25 to 30N/mm2 , it was recommended that the use of concrete made with these aggregates be restricted to lintels and columns of load bearing walls of not less than three storey to avoid excessive waste of cement and risk of building failure.

Keywords: Mathematical model, Concrete, Umuaga and Gravel Piths

Background to the Study
Umuaga, a town in the south eastern part of Nigeria, is a popular place for collection of high quality sedimentary rock aggregates for mainly building construction. These aggregates are produced in variety of sizes and are all classed as high quality; but 8-16mm size aggregate is highly contaminated with clay and silt and its quality doubtful.
This report seeks to study the quality of this aggregate (8-16mm size) through a mathematical model for the strength of concrete made from it and compare it with that of other known good aggregates like granite to enable boundaries for its use as an aggregate for structural concrete to be specified to guide stake holders.

The mathematical model is to be derived from scheffe's simplex method of mixture design, popular in the field of Industrial and Chemical Engineering. In this method of optimization only the proportions of the components in the mixture are required to study a given property of the mixture. Concrete being a mixture, this method can also be applied to it. Firstly a simplex is defined as a convex polyhedron with (k+1) vertices produced from k intersecting hyper planes in k-dimensional space (Akhnazarova,1982). Any co-ordinate system above 3-dimensions is referred to as hyper planes; such planes are not orthogonal. A 2-dimentional regular simplex is therefore an equilateral triangle, while a 3-dimentional regular simplex is a regular tetrahedron.

To describe a response surface for the prediction of a mixture property, for mixtures consisting of several components q ,Scheffe used a regular (q-1) simplex to achieve it (Scheffe, 1958). Following the definition of simplex already explained , if q=2, the required simplex is a straight line. For q=3, the required simplex is an equilateral triangle, and if q=4 the simplex is a regular tetrahedron, etc. For such multi component system , the response surface is normally described using a high degree polynomial, of the type in Eq 1.0, having number of coefficients given by C^{n}_{a+n} where n is the degree of the polynomial(Zivord,2004).

$$\hat{y} = b_o + \sum_{1\le i\le q} b_i x_i + \sum_{1\le i<j\le q} b_{ij} x_i x_j + \sum_{1\le i<j<k\le q} b_{ijk} x_i x_j x_k + \sum b i_1 i_2 \ldots i_n x i_1 x i_2 \ldots x i_n \quad ------------------- \quad ----------- (1.0)$$

Knowing that Eq (2.0) also holds for mixtures,

$$\sum_{i=1}^{q} x_i = 1 \quad ------------------------------(2.0)$$

Where xi $\geq$ 0 represents the component concentration in the mixture , Scheffe (1958) was able to reduce the number of coefficients in Eq1.0 to arrive at a new polynomial whose number of coefficients is given by C^{n}_{q+n-1} ,thereby reducing the number of experimental trials required to evaluate the coefficients. Demonstrating this reduction for a four-component mixture we have:
From Eq (1.0) and Eq (2.0)

$$\hat{y} = b_o + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + +b_{12}x_1x_2 + b_{13}x_1x_3$$
$$+b_{14}x_1x_4 + b_{23}x_2x_3 + b_{24}x_2x_4 + b_{34}x_3x_4 + b_{11}x_1^2 + b_{22}x_2^2$$
$$+b_{33}x_3^2 + b_{44}x_4^2 \quad - \quad - \quad - \quad - \quad -- \; --------- 3.0)$$
$$and x_1 + x_2 + x_3 + x_4 = 1 - \quad - \quad - \; ---------------- \; (4.0)$$

Multiplying Eq (4.0) by bo, x1, x2, x3 and x4, separately, and rearranging the variables the following equations are obtained;

$$b_o = b_ox_1 + b_ox_2 + b_ox_3 + b_ox_4 \; - - ---------------------(5.0)$$
$$x_1^2 = x_1 - x_1x_2 - x_1x_3 - x_1x_4 \; - - - - \; (6.0)$$
$$x_2^2 = x_2 - x_1x_2 - x_2x_3 - x_2x_4 \; - - - - \; (7.0)$$
$$x_3^2 = x_3 - x_1x_3 - x_2x_3 - x_3x_4 \; - - - - \; (8.0)$$
$$x_4^2 = x_4 - x_1x_4 - x_2x_4 - x_3x_4 \; - - - - \; (9.0)$$

Substituting Eqs 5.0, 6.0, 7.0, 8.0 and 9.0 into Eq. 3.0 and rearranging yields

$$\hat{Y} = \beta_1x_1 + \beta_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_{12}x_1x_2$$
$$\beta_{13}x_1x_3 + \beta_{14}x_1x_4 + \beta_{23}x_2x_3 + \beta_{24}x_2x_4$$
$$+\beta_{34}x_3x_4 \quad - \quad - \quad - \quad - \quad - \quad ---------------(10)$$

Eq(10) is the scheffe's reduced second degree polynomial for 4-component mixtures. It has only 10 coefficients instead of 15, reducing the number of experimental trials by 5.

Factor Notation on a Simplex Lattice

Each component to be used in a mixture is divided into (n+1) similar level (parts), where n is the degree of the polynomial to be used in the model. The component compositions and their respective concentrations in each mixture are shown by the use of subscripts. For example, a mixture xij could contain only one component with its full concentration denoted as x1, x2, x3 or x4 ; another mixture could contain two components of equal concentrations denoted as x12, x13, x14, x23, x24 or x34. A mixture having two components of different concentrations is denoted as x112, x113, x224, etc. – the number of times each of the components appears in the subscript relative to the other is a measure of their relative concentration.

These mixtures are arrayed on the simplex to form a lattice, i.e. a uniform scatter that could be joined by crossing straight lines parallel to the edges of the simplex. For tetrahedrons, for instance, starting from the vertex with straight component mixtures x1, x2, x3, etc; followed by the edges with binary component mixtures x12, x13, x24, etc; then the faces with 3-component mixture x124, x234 etc; and finally the interior with 4-component mixtures, this sequence is followed until all the required experimental trials are depicted on the simplex. Fig. 1.0 shows the positions of all the factors (mixtures) on a regular tetrahedron for a second degree polynomial to be used for the description of the response space for a 4-component mixture – a (4, 2) – lattice.

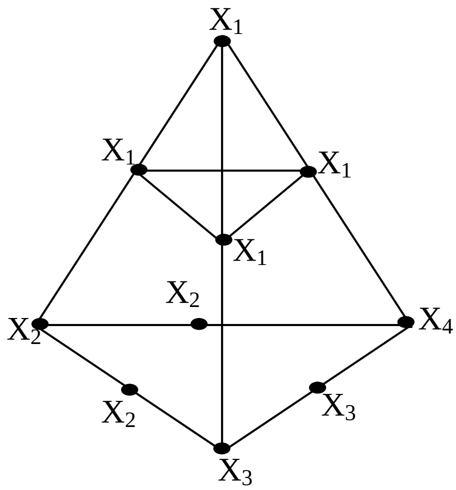

Fig. 1.0: Factor Notations for a (4, 2) – Lattice

A matrix table is normally used to display these factors (see left side of table 1.0) each row displaying a mixture with its components and concentrations.

Table 1.0: Matrix table for Scheffe's (4, 2) - Lattice Polynomial

Pseudo - Components | Response - Real Components

S/N	x_1	X_2	X_3	X_4		Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.6	1.0	1.5	4
2	0	1	0	0	Y_2	0.5	1.0	1.0	1½
3	0	0	1	0	Y_3	0.55	1.0	1½	3.0
4	0	0	0	1	Y_4	0.555	1.0	2 ½	4.0
5	½	½	0	0	Y_{12}	0.55	1.0	1.25	2.75
6	½	0	½	0	Y_{13}	0.575	1.0	1.5	3.5
7	½	0	0	½	Y_{14}	0.578	1.0	2.0	4.0
8	0	½	½	0	Y_{23}	0.525	1.0	1.25	2.25
9	0	½	0	½	Y_{24}	0.528	1.0	1.75	2.75
10	0	0	½	½	Y_{34}	0.553	1.0	2.0	3.5

For the fact that concrete mixtures have its sum of proportions above unity a congruent simplex is necessary such that the mix proportions at the vertices show the range of w/c ratio, cement, fine aggregate and coarse aggregate ratios, respectively, the required polynomial model will cover or predict (see fig. 2.0).

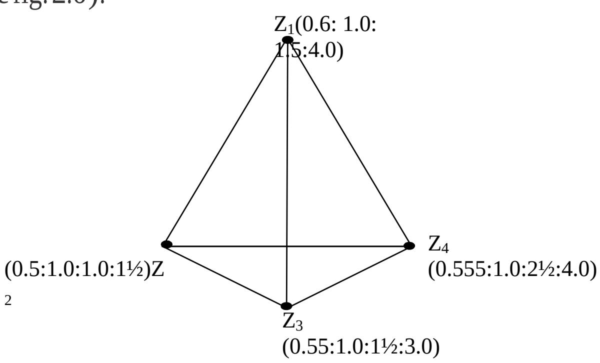

Fig. 2.0: Real Component Simplex (only vertices are shown)

The former simplex, fig. 1.0, is called Pseudo-component simplex and the later, fig. 2.0, real component simplex. From the later (real components) a Z-matrix is formed whose transpose becomes the conversion factor from pseudo to real component; i.e. from fig. 2.0

$$z = \begin{bmatrix} 0.6 & 1.0 & 1.5 & 4.0 \\ 0.5 & 1.0 & 1.0 & 1\frac{1}{2} \\ 0.55 & 1.0 & 1\frac{1}{2} & 3.0 \\ 0.555 & 1.0 & 2\frac{1}{2} & 4.0 \end{bmatrix}$$

$$and Z^T = \begin{bmatrix} 0.6 & 0.5 & 0.55 & 0.555 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.5 & 1.0 & 1\frac{1}{2} & 2\frac{1}{2} \\ 4.0 & 1\frac{1}{2} & 3.0 & 4.0 \end{bmatrix} \quad - \quad - \quad - \text{-------------} 11$$

To demonstrate the use of Eq (11) in table 1.0, the 5th row in the real component side is obtained by multiplying [Z]T matrix by the corresponding row in the pseudo-component side of table 1.0, i.e.

$$\begin{bmatrix} 0.6 & 0.5 & 0.55 & 0.555 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.5 & 1.0 & 1\frac{1}{2} & 2\frac{1}{2} \\ 4.0 & 1\frac{1}{2} & 3.0 & 4.0 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.55 \\ 1.0 \\ 1.25 \\ 2.75 \end{bmatrix}$$

In this way all the corresponding rows in the real component side are obtained producing a congruent table and simplex suitable for concrete.

Material and Method

(i) Materials

Materials needed for the experiment include sample of unwashed coarse aggregate (gravel, 8-16mm size only) from Umuaga gravel pith. The samples were stored in sacks, indoor, so that moisture variation in the samples would be minimal. The laboratory equipments needed include universal crushing machine, 150 x 150mm x 150mm cube moulds, mould oil, weighing balance, trowel and curing tank.

(ii) Method

Using the weighing balance; water, cement fine aggregate and coarse aggregate were weighed out, respectively, in the proportions shown in table 1.0- right side - in such a way that the materials weighed out served for three cubes. The materials were thoroughly mixed together inside a non-absorbent container before water was added and final mixing was done. Three cubes were cast from each of the mix proportions, making 60 cubes in the whole. The fresh concrete was filled into the moulds in three layers, each layer tamped not less than 25 times. The top was scraped off with the trowel. The concrete was allowed to harden for 24 hours, after which the mould was removed and the cubes were water-cured for 28 days in the curing tank. At the end of 28 days the cubes were crushed in the universal crushing machine. The results and averages from the test points were tabulated in columns 7 to 10 of table 2.0. Extra ten test points were provided for validation of the model.

S/N	Pseudo-Components				Response Symbols	Replicate Responses (N/mm^2)			Average Response $\bar{Y} N/mm^2$	Predicted values $\hat{Y} N/mm^2$	Real Components (Concrete Mix ratios)			
	X_1	X_2	X_3	X_4		1	2	3						
1	1	0	0	0	Y_1	4.44	4.44	3.57	4.15	4.15	0.6	1	1 ½	4
2	0	1	0	0	Y_2	8.89	8.89	8.00	8.59	8.59	0.5	1	1	1 ½
3	0	0	1	0	Y_3	5.56	4.44	6.22	5.41	5.41	0.55	1	1 ½	3
4	0	0	0	1	Y_4	5.33	4.44	4.00	4.59	4.59	0.555	1	2 ½	4
5	$1/2$	$1/2$	0	0	Y_{12}	8.44	6.22	7.78	7.48	7.48	0.55	1	1¼	2¾
6	$1/2$	0	$1/2$	0	Y_{13}	8.89	11.11	8.22	9.41	9.41	0.575	1	1½	3 ½
7	$1/2$	0	0	$1/2$	Y_{14}	8.44	10.44	10.67	9.85	9.85	0.578	1	2	4
8	0	$1/2$	$1/2$	0	Y_{23}	11.33	10.67	8.00	10.00	10.00	0.525	1	1 ¼	2¼
9	0	$1/2$	0	$1/2$	Y_{24}	7.78	11.56	14.89	11.41	11.41	0.528	1	1¾	2¾
10	0	0	$1/2$	$1/2$	Y_{34}	10.89	10.67	10.44	10.67	10.67	0.533	1	2	3½
							Control Points							
11	$1/2$	0	$1/4$	$1/4$	C_1	10.67	8.00	10.22	9.63	11.05	0.576	1	2 ¾	3 ¾
12	$1/4$	0	$1/2$	$1/4$	C_2	11.11	11.11	6.67	9.63	11.41	0.564	1	1¾	3½
13	$1/4$	$1/4$	$1/4$	$1/4$	C_3	11.11	13.11	9.33	11.18	11.86	0.551	1	1.625	3.125
14	$2/3$	0	0	$1/3$	C_4	11.00	9.00	10.89	10.30	9.17	0.585	1	1.833	4.0
15	$1/4$	$1/4$	$1/2$	0	C_5	10.22	9.78	8.89	9.63	9.98	0.55	1	1.375	2.875
16	$1/4$	$1/2$	0	$1/4$	C_6	9.78	17.78	12.44	13.33	10.82	0.539	1	1 ½	2 ¾
17	$1/4$	0	$1/4$	$1/2$	C_7	9.78	11.11	9.11	10.00	11.42	0.535	1	2	3 ¾
18	$1/2$	$1/4$	0	$1/4$	C_8	9.78	9.42	7.56	8.92	9.87	0.564	1	1.625	3.375
19	$1/4$	$1/2$	$1/8$	$1/8$	C_9	7.78	8.89	12.22	9.63	10.71	0.538	1	1.375	2.625
20	$1/3$	$1/3$	0	$1/3$	C_{10}	8.89	8.89	10.22	9.33	10.85	0.552	1	1.667	3.167

Development of the Model

The general form of Scheffe's (4,2) – Lattice Polynomial is given by

$$\hat{Y} = \sum_{1\le i\le 4} \beta_i x_i + \sum_{1\le i<j\le 4} \beta_{ij} x_i x_j - \quad - \quad - \quad - \quad 12$$

where $\beta_i = \bar{y}_i$, $\beta_{ij} = 4\bar{y}_{ij} - 2\bar{y}_i - 2\bar{y}_j$

From table 2.0, Column 10:

$\beta_1 = 4.15, \beta_2 = 8.59, \beta_3 = 5.41, \beta_4 = 4.59$

$\beta_{12} = 4 \times 7.48 - 2 \times 4.15 - 2 \times 8.59 = 4.44$

$\beta_{13} = 4 \times 9.41 - 2 \times 4.15 - 2 \times 5.41 = 18.52$

$\beta_{14} = 4 \times 9.85 - 2 \times 4.15 - 2 \times 4.59 = 21.92$

$\beta_{23} = 4 \times 10.00 - 2 \times 8.59 - 2 \times 5.41 = 12.00$

$\beta_{24} = 4 \times 11.41 - 2 \times 8.59 - 2 \times 4.59 = 19.28$

$\beta_{34} = 4 \times 10.67 - 2 \times 5.41 - 2 \times 4.59 = 22.68$

The model for compressive strength for Umuaga sample becomes

$$\bar{Y} = 4.15x_1 + 8.59x_2 + 5.41x_3 + 4.59x_4 + 4.44x_1x_2 + 18.52x_1x_3 + 21.92x_4 + 12.00x_2x_3 + 19.28x_4 + 22.68x_3x_4$$
$$- - - - - - - - - 13$$

The predictions from Eq 13 are given in table 2.0 Column 11.

Validation of the Model (Test for Adequacy)

Adequacy of the model (Eq 13) can be tested through Fisher's variance ratio, whereby the calculated value of Fisher's ratio F is compared with the tabulated value in the Quantile of the F-Distribution.

$$F = \frac{S_g^2}{S_e^2} \quad - \quad - \quad - \quad - \; - \quad 14$$

$$\text{Where } Sg^2 = \frac{m}{n-1}\sum_{i=1}^{n}(\bar{y}_i - \hat{y}_i)^2 - \; - \; - 15$$

$$\text{and } S_e^2 = \frac{1}{n(m-1)}\sum_{i=1}^{n}\sum_{u=1}^{n}(y_i - \bar{y}_i)^2 \quad - \; - \quad - \; - \quad 16$$

In the above equations n is the number of experimental trials, m is the number of replications for each experimental trial, $\bar{y}_i$ is the number of coefficients in the model, is the average response for i[th] experimental trial, y^{iu} is the u[th] replicate response value for ith trial. If F is less than the tabulated value, then the model is adequate, i.e.

$$S_g^2 = \frac{3}{10} \times 19.7424$$

$$S_e^2 = \frac{1}{40} \times 120.8724$$

$$F = \frac{S_g^2}{S_e^2} = \frac{(^3/_{10}) \times 19.7424}{(^1/_{40}) \times 120.8724}$$

$$= 1.96 < 2.1 \;\; okay$$

where the value 2.1 is the limiting value of F obtained from any table of Quantiles of F-Distribution.

Optimization of the Model

The model (Eq 13) was optimized through a Quick-Basic computer program, whose flowchart is given in Fig. 3.0. The maximum values given by the computer for strength, water, cement, fine aggregate and coarse aggregate ratios are 12.2308KN/mm2, 0.532, 1, 1.6 and 2.7, respectively.

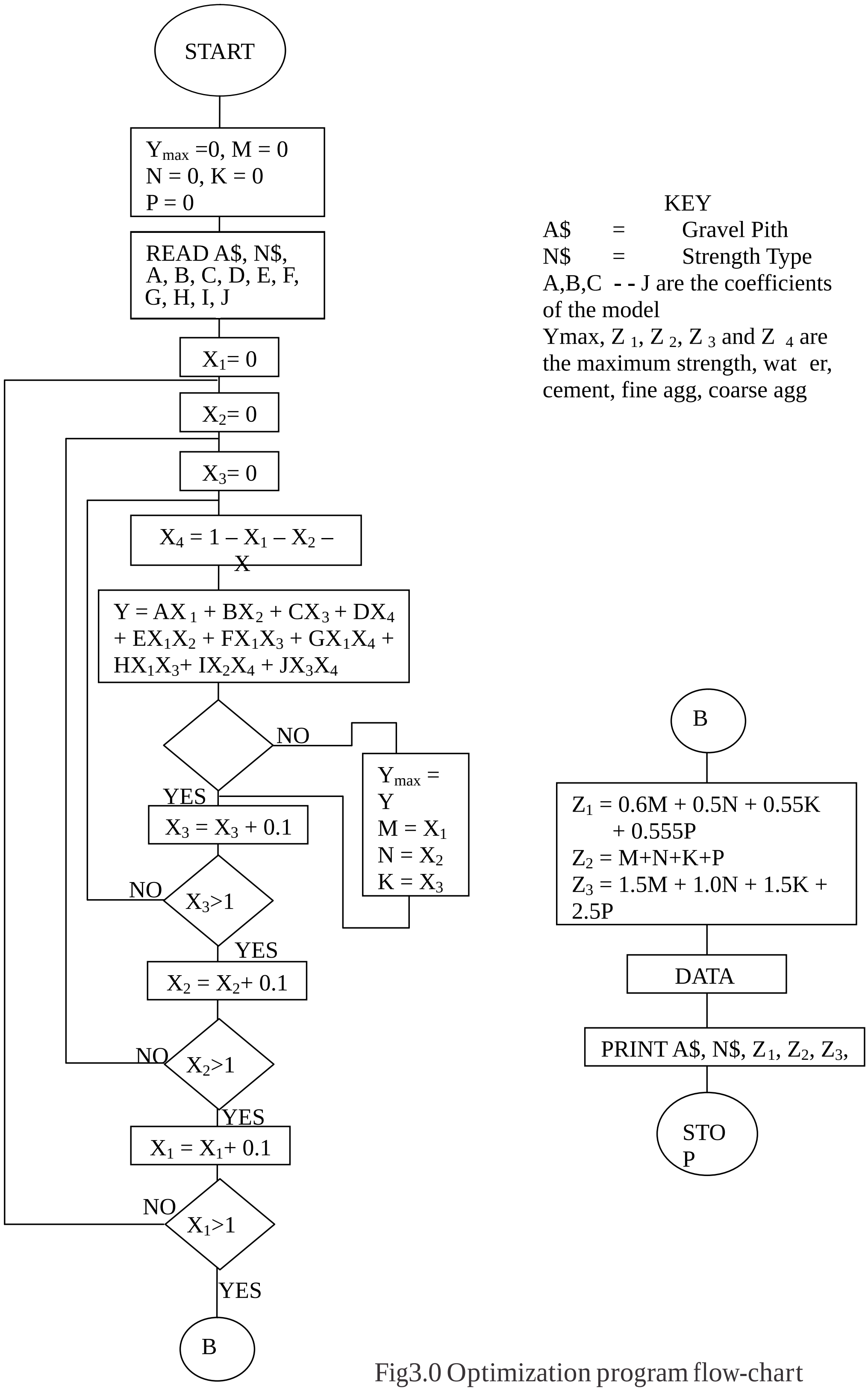

Fig3.0 Optimization program flow-chart

Discussion of Results

Looking at the results of experiments and predictions from the model in table 2.0 (Columns 7, 8, 9, 10 and 11) it can be observed that the compressive cube strengths for the various mix proportions are clearly very low. Considering the optimum value given by the computer program, whose proportions are comparable with that of grade 25 concrete, when granite is used instead - it has an average strength of 12.2308KN/mm2, which is about half of the expected value - this shows that aggregates of sizes 8 – 16mm from Umuaga are inferior.

Recommendation and Conclusion

From the above results and discussions it is obvious that concretes from these aggregates cannot be used in areas where there is excessive compressive and tensile forces such as bridges, culverts, thin slabs, foundation, etc. It is therefore recommended for only columns and lintel of load bearing walls, the storey should not be greater than three.

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