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Distribution and Inventory Planning in Multi-Echelon Supply Chains Under Demand Uncertainty

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Abstract

Distribution and inventory planning in a multi-echelon system are studied under an uncertain demand context. To deal with this problem a mixed integer linear programming (MILP) model is proposed. This considers a multi-echelon system formed by N-warehouses and M-retailers. The problem consists on determining the optimal reordering plan for the operating network, which minimises the overall system's operation cost. The uncertain demand faced by retailers is addressed by defining the optimal safety stock that guarantees a given service level at each regional warehouse and each retailer. Also, the risk pooling effect is taken into account when determining inventory levels in each entity. A case study based on a real retailer distribution chain is presented and solved.

Keywords*: Supply chain management; Inventory planning; Mixed integer linear programming; MILP; Guaranteed service approach; Demand uncertainty; Risk pooling*

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Background to the Study

The distribution operation within different industries faces uncertainties that cover a wide range of factors such as demands, prices and lead times for the supply of products. Triki and Al-Hinai (2016) researched the optimisation techniques for multi-period planning horizon and Omrani and Ghiasi (2017) studied optimisation problems with data uncertainty. Demand uncertainty and bullwhip effect phenomenon are important drivers that all managers must to consider (Vicente et al., 2018). Between these, demand uncertainty may well have the dominant impact on profits and service level. This can lead to excess inventories or inability to meet service level. Excess inventory results in unnecessary holding costs, while the inability to meet the customer needs results in both loss of profit with the possibility, on long-term, loss of customers (Jung et al., 2004).

Inventory optimisation in a multi-echelon supply chain network, characterized by an uncertain demand, is a real-world problem (Amiri-Aref et al., 2018). In this context, optimal inventory planning has become a major goal of the companies in order to simultaneously reduce costs and improve service level in today's increasingly competitive business environment (Daskin et al., 2002; Axsater, 2003; Yadollahi et al., 2017). A high service level can be obtained by maintaining increased inventory levels to hedge against demand uncertainties. Although additional inventory improves service level, it increases inventory holding cost. It is then necessary a trade-off between service level and inventory holding cost. This can be achieved through the solution of stochastic optimisation problems where the inventory levels are the key optimisation variables (Stephan et al., 2010). One such approach implies the use of the safety stock as a lower bound on the inventory level which is chosen such as to absorb some level of the demand uncertainty (Graves and Willems, 2000).

There exists a large number of works on estimating safety stock levels based on classical inventory theory. However, they fail to address the key features of realistic supply chain problems, namely, multiple products sharing multiple facilities with capacity constraints and demands originating from multiple customers. In addition, in real world supply chains, safety stock levels are dependent on factors such as the probabilistic distribution of the demands, the demand to capacity ratio, service level on meeting the demands for multiple products and transportation lead times among facilities. Such factors introduce complexities that classical inventory models simply do not accommodate (Porteus, 2002; Chopra and Meindl, 2004).

The main objective of this paper is to explore this opportunity by adapting the concept of safety stock into a network inventory planning model. Within this context, the goal of the present research is to develop a model that includes lower bounds on the inventory levels of various products and through different entities. Additionally, the approach entails the definition of the safety stock as a model variable and a guaranteed service level as a model parameter to reduce the shortage in inventory levels. The model also considers risk pooling effect, first referred by Eppen (1979), which states that significant safety stock cost can be saved by grouping in one central location the demand of multiple stocking locations. The remainder of this paper is organised as follows. Section 2 includes a literature review on mathematical optimisation approaches to model demand uncertainty and the guaranteed service approach to model the multi-echelon distribution and inventory planning system. The problem definition is given in Section 3. Section 4 describes the distribution and inventory planning mathematical model. The case study is present in Section 5. Section 6 presents the results and analysis. Finally, the conclusions are drawn in Section 7.

Literature Review

Mathematical optimisation approaches applied to the modelling of inventory planning in supply chains considering uncertain demand has been researched over the last years, but the inventory management is usually considered without detailed inventory planning supply chain policies (Inderfurth, 1991; Minner, 2001; Simchi-Levi and Zhao, 2011; Hu et al., 2017). O'Driscoll (2017) proposed a two-stage stochastic programming model for a competitive oil refinery with stochastic crude and fuel prices. Some research revealed that the nature of demand uncertainty was the key differentiator between the various supply chain optimisation techniques (Cole and Bradshaw, 2016; Zaman and Saha, 2018). In the published models, the safety stock is often given as a parameter and it usually is treated as a lower bound of the total inventory level (Relvas et al., 2006; Schulz et al., 2005; Paterson et al., 2011). This approach cannot optimize the safety stock levels, especially when considering demand uncertainty. Thus, it can only provide an approximation of the inventory cost and may lead to suboptimal solutions. Jung et al. (2004) use a simulation-optimisation framework to determine the optimal safety stocks levels of a supply chain with consideration of production capacity.

On the other hand, most of the existing literature focuses on single-echelon systems. The uncertain demand is addressed by defining the optimal amount of safety stock that guarantees certain service level at a given customer. Daskin et al. (2002) introduced a model in which supply chains design decisions integrate inventory considerations. It is assumed that no limitation in storage capacity is considered and all lead times from supplier to distribution centres are the same. Thus, given these assumptions, the inventory structure is considered as a single-echelon system. Similar research can be found in Shen et al. (2003). Ozsen et al. (2008, 2009) extend the model of Daskin et al. (2002) and Shen et al. (2003) to include capacities on the inventory held. Bossert and Willems (2007) extend the guaranteed service modelling framework in order to optimise the inventory policy in a supply chain.

The first one uses a stochastic programming model where uncertainty is considered directly using a scenario-based approach (Tsiakis et al., 2001; Sahinidis, 2004). Each scenario is associated with a certain probability of occurrence and represents one possible realisation for the uncertain parameter. In general, two decision stages are considered. In the first stage, 'here and now' decisions have to be made before the uncertain parameter realisation is known. In the second stage, 'wait and see' decisions are considered which are associated with a recourse action because they can be made after the random parameter is known. The main disadvantage of this method is that the model size tends to increase rapidly with the number of scenarios considered. In addition, it is not always feasible to explicitly enumerate all possible discrete values of the uncertain parameter.

The second one consists of using the chance constraint approach in which each uncertain parameter is treated as a random variable with a given probability distribution, which is applied in several cases to model demand uncertainty (Gupta and Maranas, 2003; You and Grossmann, 2008; Rodriguez and Vecchietti, 2011; Humair and Willems, 2011). The guaranteed service approach aims at determining the optimal placement and amount of safety stocks in a multi-echelon system to ensure the overall target service level at the lowest cost (Eruguz et al., 2014). Recently, Hong et al. (2018) study a supply chain configuration problem to optimise the service time and option selection decisions to minimise the overall cost of the supply chain. Generally, in a supply chain, most of the parameters are not deterministic, for this reason is better to consider demand and service time as uncertain parameters (Rashid et al., 2018). When applying this approach demand uncertainty is considered by specifying a demand level above the mean that must be satisfied. One strategy explored by You and Grossmann (2008) is to define the safety stock as a decision variable and a guaranteed service level as a parameter in the model to reduce the shortage in the inventories.

In this work, the second approach is chosen as it allows determining a safety stock level at supply chain entities in order to guarantee a certain service level and avoids the creation of multiple scenarios in a single model, which increases largely the model size. The guaranteed service approach has been addressed in several problems in multi-echelon stochastic inventory planning and supply chain optimisation (Eruguz et al., 2016) but it has not yet been treated on short-term inventory planning problems (Graves and Willems, 2000, 2005, 2008; Neale and Willems, 2009). On another hand, integrating stochastic inventory planning into the operational planning supply chain is nontrivial, and it has not been addressed in the existing literature.

The concept of guaranteed service approach, which is used in this work is based on the works by Graves and Willems (2000) and You and Grossmann (2010, 2011). Such concept is here applied to multi-echelon networks where when comparing to single-echelon inventory must consider explicitly the presence of lead time, which may include material handling time and transportation time. Within single-echelon systems, the ones already addressed in the literature, lead time is exogenous and generally can be treated as a parameter. The research objectives of the paper were presented and an adequate methodology is then required. To this end, the guaranteed service approach for multi-period, multi-product and multi-echelon supply chain is applied to the problem in next section.

Problem definition

This section is an extended version that complements a previous one presented in a conference by Vicente et al. (2015). A generic supply chain under product demand uncertainty is considered in this study, across the guaranteed service model approach. It comprises one central warehouse, multiple regional warehouses and multiple retailers as depicted in Figure 1, where multiple products are distributed over a given time horizon of multiple time periods.

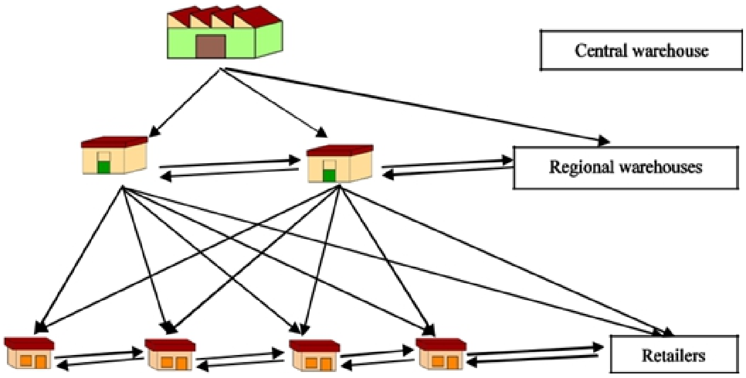

Figure 1: Supply chain structure (see online version for colours)

The structure assumes that retailers replenish their inventories from the regional warehouses, these replenish their inventories from a central warehouse and customer demand is observed at the retailers. Each retailer faces a normally distributed demand with mean μ and variance σ^2, which is independent of the other retailers' demands. For this supply chain structure, single sourcing is assumed, e.g., each retailer can only be served by one regional warehouse as each regional warehouse can only be served by one warehouse (central warehouse). Lateral transshipment between regional warehouses and between retailers is allowed.

The corresponding deterministic order processing times, which include the material handling time and transportation time, are given. The guaranteed service time of the central warehouse and the guaranteed service time of each retailer are known. The safety stock factors for regional warehouses and retailers are also given. All storage and transportation capacities are limited and transportation occurs after orders have been placed. If the demand in a given time period and at a given retailer is not satisfied, this is assumed as a lost sale. The first and second ones (stock and safety stock) are defined by unit stored and by time period on each regional warehouse or retailer. The third ones are defined by unit of product transported and are dependent of the order processing time.

Transportation costs are considered by unit of material transported between the different stages of the supply chain. Related to these are the transshipping costs that represent the lateral transportation costs by unit that occurs within each stage between two identical entities. These can occur between regional warehouses or between retailers. Finally, lost sales costs are associated to the demand that cannot be satisfied and are defined by unit of product.

The problem in study can then be defined as follows:

Given

i. The planning time horizon and the defined discrete time scale.
ii. The number of regional warehouses and retailers.
iii. The number of products.
iv. Initial inventory by product in each regional warehouse and retailer.
v. Mean and standard deviation of demand for each product on a time period basis (the product demand is normally distributed and occurs in retailers).
vi. Storage capacities in each regional warehouse and retailer by time period.
vii. Transportation capacities between entities.
viii. Order processing time between entities
ix. Ordering costs by order of each product at each regional warehouse and retailer (independent of order quantity).

Determine

i. The inventory profiles by product throughout the planning time horizon at each regional warehouse and retailer in each time period.
ii. Safety stock by product for the planning time horizon in each regional warehouse and retailer.
iii. The flows of products across the supply chain for each time period. These involve shipping quantities between entities on different supply chain levels and transshipment quantities between entities on the same supply chain level.
iv. Lost sale quantities by product at each retailer in each time period.

Distribution and Inventory planning Mathematical Model

The supply chain distribution and inventory planning problem presented is formulated as a MILP model, as an extended version that complements a previous one presented in a conference by Vicente et al. (2015). This model uses a variable order quantity covering the demand of variable length time periods. It considers time represented through a discretized time scale, where the time periods have equal durations. The indices, sets, parameters and variables (non-negative continuous and binary) used in the model formulation are defined using the following notation

Case Study

In this section we present a case study based on a retail company. Due to confidentiality reasons the data provided has been changed but still describes the real operation.

The model was implemented in GAMS 24.2 modelling language and solved using CPLEX 12.3 solver in an Intel Core i7 CPU 3.40 GHz and 8GB RAM. The stopping criteria were either a computational time limit of 3,600 seconds or the determination of the optimal solution.

Table 1: General case study parameters

Parameters	*Values (€)*
Ordering cost ($OCij, j \in DN, \forall i$)	20
Holding cost ($HOCij, j \in W, \forall i$)	0.2
Holding cost ($HOCik, k \in R, \forall i$)	0.6
Holding in transit cost ($HTCi0j, j \in W, \forall i$)	0.3
Holding in transit cost ($HTCijl, j \in W, l \in W, j \neq l, \forall i$)	0.3
Holding in transit cost ($HTCijk, j \in W, k \in R, \forall i$)	0.9
Holding in transit cost ($HTCikm, k \in R, m \in R, k \neq m, \forall i$)	0.9
Lost-sales cost ($LSCikt, k \in R, \forall it$)	25

Table 2: Unitary products transportation costs (euro)

		Warehouse 1	*Warehouse* 2	*Retailer* *1*	Retailer 2	*Retailer* 3	*Retailer* *4*
TRC	Warehouse 0	0.55	0.22	0	0	0	0
	Warehouse 1	0	0.7	0.22	0.2	0.32	0.38
	Warehouse 2	0.7	0	0.68	0.52	0.34	0.1
	Retailer 1	0	0	0	0.2	0.8	1.3
	Retailer 2	0	0	0.2	0	0.3	1.0
	Retailer 3	0	0	0.8	0.3	0	0.36
	Retailer 4	0	0	1.3	1.0	0.36	0

Table 3: Initial inventory level (Ito) on warehouses and retailers (unit)

		Warehouse 1	*Warehouse* 2	*Retailer* 1	*Retailer* *2*	*Retailer* 3	*Retailer* 4
Ito	Product 1	45	30	24	22	20	18
	Product 2	15	11	16	14	12	10
	Product 3	11	9	8	4	6	9

Table 4: Product demand parameters (PD) data for product 1/product 2/product 3 (unit)

		Mean demand	*Standard deviation*
PD	Retailer 1	12/8/6	4/4/4
	Retailer 2	11/7/9	4/4/4
	Retailer 3	10/6/7	3/3/3
	Retailer 4	9/5/6	3/3/3

Table 5: Order processing time (T1) of all products (time period)

		Warehouse 1	*Warehouse 2*	*Retailer 1*	*Retailer 2*	*Retailer 3*	*Retailer 4*
T1	Warehouse 0	2	1	0	0	0	0
	Warehouse 1	0	1	1	1	1	1
	Warehouse 2	1	0	2	2	1	0
	Retailer 1	0	0	0	1	1	1
	Retailer 2	0	0	1	0	1	1
	Retailer 3	0	0	1	1	0	1
	Retailer 4	0	0	1	1	1	0

The supply chain considered involves one central warehouse, two regional warehouses and four retailers. Three main types of product families are considered. The safety stock factors (*SSFij*) for regional warehouses and retailers were considered the same and equal to 1.96, which corresponds to 97.5% service level considering that the product demand is normally distributed. This service level is common in the industry sector of the company in study. The guaranteed service time of the central warehouse (*SIi*0) is 1 time period. As the last echelon, representing the retailers, is an exogenous input (which can be treated as a parameter), the guaranteed service time of retailers (*Rikt*) are set to 0 in order to have an immediate response. The maximum storage capacity for warehouses is of 5,000 units and for retailers is of 500 units. The transportation quantity limit between entities is considered between 0 and 500 units. A seven time period planning horizon was assumed to test our model (modelled in Section 4), which uses a variable order quantity covering the demand of variable length time periods. Tables 1 to 5 present the parameters' values considered for this case study, including general model parameters, transportation costs, initial inventory levels, product demand and order processing time. Customer demands at retailers are random values of the normal distribution (Table 4), generated in GAMS 24.2 modelling language.

Results and Analysis

The retail company wants to compare two options for order management in the supply chain:

Option A: Regional warehouse order and retailer order fulfilment flows per product can be formed by several flows of that product from any entity of the supply chain (e.g., on a per product perspective, each retailer could be served by any of the regional warehouses (a single combination) and by all others retailers).

Option B: Regional warehouse order and retailer order fulfilment flows by product are formed by only one flow of that product from only one entity in a different echelon of the supply chain (e.g., on a per product perspective, each retailer is only served by one regional warehouse and transshipment is not allowed). After analysing the obtained results it can be said that the company should decide by operating under option A. This option presents lower total costs and a higher service level. Although option B presents zero holding in transit and transportation transshipment costs, however it presents high lost sale costs.

Conclusions and Recommendation

This paper addresses an inventory planning model to determine the optimal inventory and distribution plan over a multi-echelon and multi-period planning time horizon under product demand uncertainty to support the decision-making process in short-term process planning. The study conclude that guaranteed service approach policy is selected to deal with uncertainty and is used to model the safety stock inventory system of a distribution company. The risk pooling effect is also considered in the model by relating the probability distribution functions of the demands in the downstream nodes to their upstream nodes and it therefore recommend: The proposed MILP model considers the safety stock level as a variable to be optimised and the service level as a parameter so as to reduce shortage occurrence in inventories.

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